Requirements for scaleable QIP

These requirements were presented in a very influential paper by David Divincenzo, and are widely used to determine if a particular physical system could potentially be used to build a *scaleable* quantum computer.

1. Existence of q-bits (i.e., tensor product structure).

2. Controllable one- and two-bit unitary gates.

3. Initializable in a known starting state.

4. Ability to measure bits in standard basis.

5. Very low intrinsic decoherence.

This week we will see how well two current experimental systems meet this list of requirements, and briefly consider other approaches.

The ion trap

One of the most powerful experimental developments of the last few decades was the development and improvement of two techniques: *laser cooling* and *electromagnetic traps*. By means of these, it is possible to cool small numbers of ions or atoms to nearly absolute zero and confine them at a precise location in an vacuum chamber, where they can be repeatedly probed by properly-tuned lasers. This is the closest we have come to being able to achieve the type of quantum measurements envisioned by von Neumann in the 1930s: projective measurements which probe the state of the system without destroying it.

Using these techniques, it may be possible to achieve all of the Divincenzo criteria. We will see in this lecture how this is done.

Atoms and resonant driving

An atom (or ion) consists of a positively charged nucleus with some number of negatively charged electrons which are bound to the nucleus by the Coulomb force. In an ion, these charges do not cancel, so the ion has a net charge; either some electrons have been stripped away or added.

Since the electrons are attracted to the nucleus, they "try" to be as close to it as possible; however, they repel each other. Also, no two electrons can be in exactly the same state due to the Pauli Exclusion Principle. The arrangement of electrons which minimizes the energy subject to these constraints is the *ground state* $|g\rangle$.

If the atom or ion acquires extra energy, an electron may be "kicked" into a higher orbit, making the atom *excited*. The lowest-lying excited state will be labeled $|e\rangle$.

One way of exciting an ion is by *resonant driving*. The principle is the same as pumping up a swing: the electrons have natural resonant frequencies. Because the electrons are charged, a periodic electric field will exert a periodic force on the electron; if the period matches the resonance frequency, the atom will be excited; it will make a transition from the initial state to a higher energy state. This kind of periodic electric field can be provided by a laser tuned to the appropriate frequency.

The proper laser frequency is determined as follows: the energy of a single photon is $\hbar\omega$, where ω is the light frequency. This energy must equal the *difference* between the two atomic levels. The atom absorbs a single photon and becomes excited. (If the laser is left on, the atom will actually make a transition back to the starting state, re-emitting the absorbed photon. This is called *stimulated emission*.)

If the laser is tuned away from a resonance frequency the rate of transitions rapidly diminishes. Because the level spacing of most atoms and ions is not very even, this means we can drive particular transitions with great specificity. For instance, it is possible to drive a transition that will happen if the atom is in state $|e\rangle$ but not in $|g\rangle$.

Also, certain transitions may be *forbidden* by other conservation laws (such as parity and angular momentum).

Q-bits and one-bit gates

We can now see how to build a q-bit out of a trapped ion. We identify the two states

$$|0\rangle \equiv |g\rangle, \quad |1\rangle \equiv |e\rangle,$$

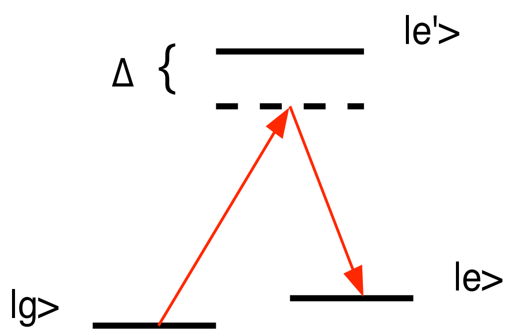
as our basis vectors, and carry out one bit gates by driving transitions with appropriately-tuned lasers.

Note that because $|g\rangle$ and $|e\rangle$ do not have the same energy, there will be a constantly-accumulating relative phase between them:

$$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|0\rangle + e^{i\Delta Et/\hbar}\beta|1\rangle.$$

We must keep track of this phase as we perform our quantum gates. In our description we will just automatically undo this phase with extra Z rotations. A description like this is called a *rotating frame*.

One complication is that for many ions, direct transitions between $|g\rangle$ and $|e\rangle$ are forbidden. (This is actually a good thing; it means that it is hard for the state $|e\rangle$ to decay.) We get around this by making the transition via a third level, $|e'\rangle$. Two lasers are used, one tuned to $|g\rangle \to |e'\rangle$, and one to $|e'\rangle \to |e'\rangle$. A photon is absorbed from the first beam, and emitted into the second.



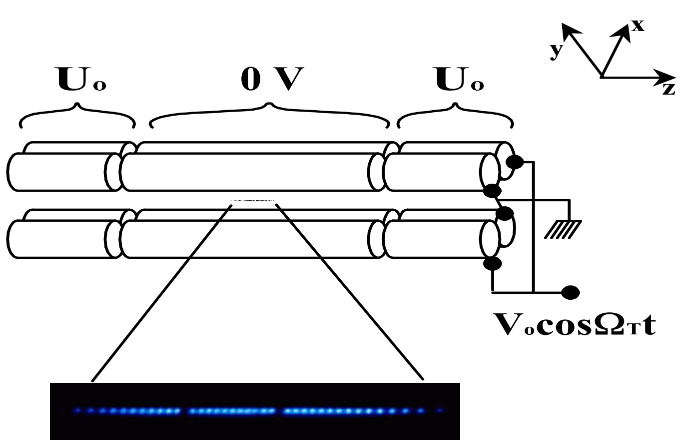
Since $|e'\rangle$ is highly excited, it is vulnerable to spontaneous emission. To avoid this, we do not tune exactly on resonance; instead, we *detune* both lasers by the same amount Δ. This slows the transition, but makes that the probability of being in $|e'\rangle$ very small.

This transition from $|g\rangle$ to $|e\rangle$ is now a two-photon process; this is called a *Raman transition.*

If we leave the lasers on for precisely the right length of time, we carry out the unitary transformation $\hat{U} = (|g\rangle\langle e| + |e\rangle\langle g|)$ in our subspace. In our computational basis, this is equivalent to an $\hat{X}$ gate. By leaving the lasers on for different lengths of time, we can do arbitrary $\hat{X}$ rotations. By driving other transitions, we can also do $\hat{Y}$ rotations; together, they are sufficient to do any one-bit gate.

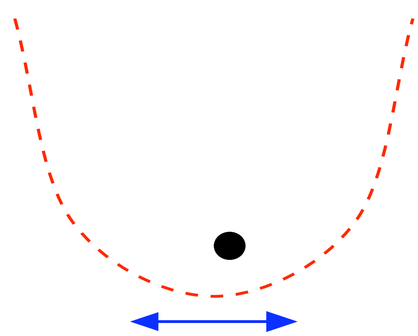
Multiple ions

It is difficult to do much interesting with only a single q-bit. To do more complicated tasks, we need to put multiple ions in the trap. Because the ions are charged, they repel each other and spread out in a line. They are widely separated enough (in principle) to be addressed individually by lasers.



Harmonic Oscillators

To produce two-bit quantum gates, we make use of an additional degree of freedom to couple the internal states of the ions: the *motion* of the ions. To understand how this works, we need to make a brief digression to talk about the simple harmonic oscillator.



The Hamiltonian of a harmonic oscillator is

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{m\omega^2\hat{x}^2}{2},$$

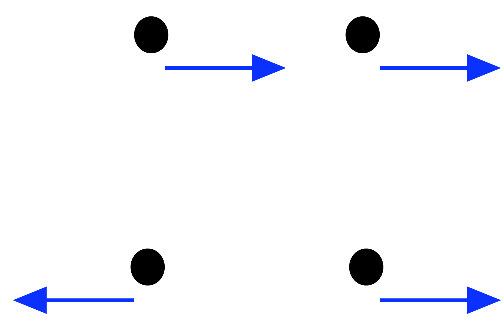
where

$$[\hat{p}, \hat{x}] = -i\hbar.$$

This system is solved by finding the energy eigenstates $|n\rangle$. These have evenly-spaced eigenvalues

$$\hat{H}|n\rangle = \hbar\omega(n + 1/2)|n\rangle \equiv E_n|n\rangle.$$

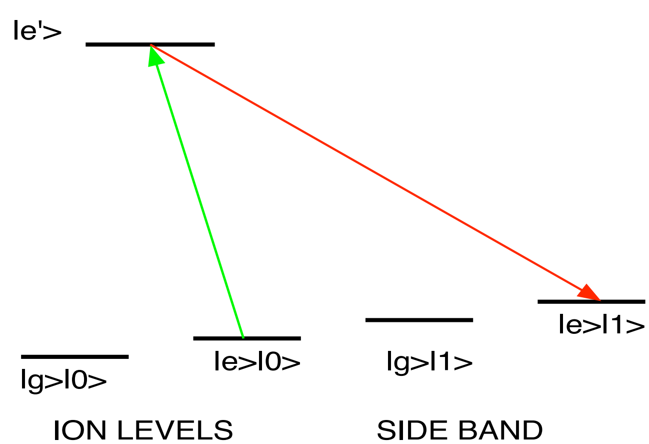
A single ion in a trap acts like a harmonic oscillator to a good approximation. What about multiple ions?



The motion of the ions can be decomposed into *normal modes*, patterns of motion like the ones illustrated above. For N ions, there are N normal modes; each of these acts like a separate harmonic oscillator with its own characteristic frequency.

By laser cooling, the ions in the trap are reduced to their *motional ground-state*: each of the normal modes is in the state $|0\rangle$. It is possible to excite transitions of one of the normal modes to an excited state by driving the ions at the resonance frequency ω.

However, there is a much more interesting possibility. It is possible to excite a normal mode *conditional on the electronic state of one of the ions.*



One can transition from the $|1\rangle$ state of the ion and ground state of the center-of-mass mode to a single excitation of the mode.

One can then use this possibility to build a controlled-$\hat{Z}$ gate between any two ions. We keep track of the states of the two ions and the center-of-mass mode by using basis states of the form

$$|\text{ion } 1\rangle|\text{ion } 2\rangle|\text{mode}\rangle.$$

The sequence of laser pulses is as follows:

1. On ion 1, drive the $|e\rangle|0\rangle \to |e'\rangle|0\rangle$ transition for a time π.

2. On ion 1, drive the $|e'\rangle|0\rangle \to |e\rangle|1\rangle$ transition for a time π.

3. On ion 2, drive the $|e\rangle|1\rangle \to |e'\rangle|0\rangle$ transition for a time 2π.

4. On ion 1, drive the $|e\rangle|1\rangle \to |e'\rangle|0\rangle$ transition for a time π.

5. On ion 1, drive the $|e'\rangle|0\rangle \to |e\rangle|0\rangle$ transition for a time π.

We see what this does to our basis states.

$$
\begin{aligned}
&|gg0\rangle \to |gg0\rangle \to |gg0\rangle \to |gg0\rangle, \\
&|ge0\rangle \to |ge0\rangle \to |ge0\rangle \to |ge0\rangle, \\
&|eg0\rangle \to |eg1\rangle \to |eg1\rangle \to |eg0\rangle, \\
&|ee0\rangle \to |ee1\rangle \to -|ee1\rangle \to -|ee0\rangle.
\end{aligned}
$$

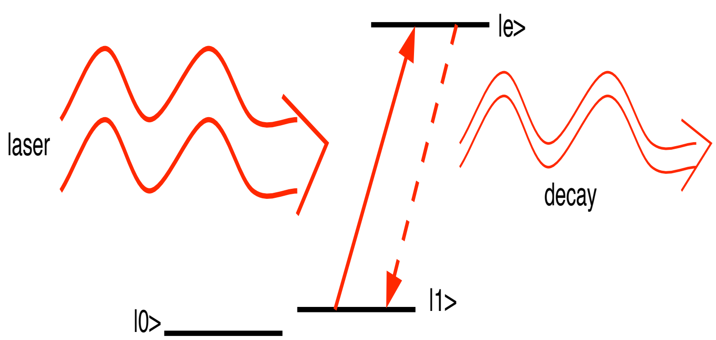
This is the controlled-Z gate between ions 1 and 2. With two Hadamards, we can make a CNOT. Together with our one-bit transitions, we have a universal set of unitary gates.

Because all of the ions participate in the normal mode, it doesn't matter whether or not the two ions are near each other. We use the motional degree of freedom as a "bus" to temporarily store one of our q-bits and have it interact with another.

Measurement

This would all be pointless if we were unable to measure the state of our q-bits. We can do this by resonant driving as well.

The key is to drive a transition from one of the basis states to an *unstable* excited state. This state will rapidly decay, emitting a photon in a random direction. This transition can be driven repeatedly, scattering many photons in a short time.



If the ion is in state $|1\rangle$ it will glow visibly when illuminated by a properly-tuned laser. If in state $|0\rangle$ it will remain dark. This is a near-perfect projective measurement.

Decoherence

These are the main intrinsic sources of decoherence for the linear ion trap:

1. *Spontaneous emission.* Since the $|e\rangle \rightarrow |g\rangle$ transition is forbidden, the $|e\rangle$ state has a long lifetime; however, by more complicated processes it can still decay. More significant is the possibility of decay from an excited state in the performance of a gate; by detuning and using metastable states, this can be kept under control.

2. *Leakage.* To act like a q-bit, the ion must remain in the subspace spanned by $|g\rangle$ and $|e\rangle$. There is always a possibility of an accidental transition to states outside this space. This is mainly controlled by tuning the lasers very precisely, and choosing ions whose transition frequencies are not too close together.

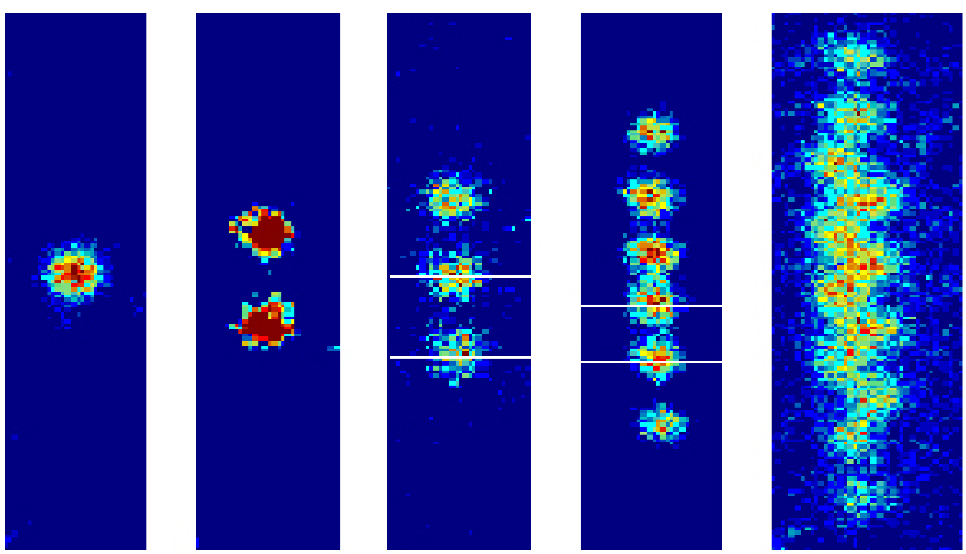
3. *Heating.* Until recently, this was the dominant source of decoherence. Because the ions are charged, they are very sensitive to the presence of stray electric and magnetic fields. These can lead the the normal modes "heating up," which interferes with two-bit gates. Recently, a great deal of progress has been made, by carefully designing the equipment and actively cooling the ions using *sympathetic cooling* of extra ions in the trap.

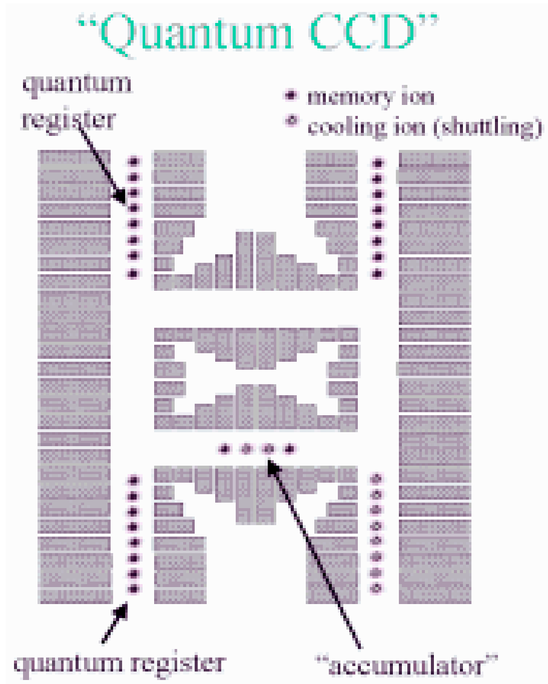
In addition to these, there are the usual problems of precision:

4. To work as described, the lasers must be very precisely tuned, and the intensity and duration of pulses tightly controlled. There are also difficulties if the lasers are not tightly enough focused on individual ions (though careful design can get around this).

Recent Developments

The scheme of packing all the ions into a single trap is inherently limited. Because only a single normal mode (or at most a few) can be used at a time, it is impossible to do many two-bit gates in parallel. Cooling must be turned off while gates are performed, which makes error rates grow. It is difficult to focus a laser down onto a single ion without accidentally affecting its neighbors. With a single trap, scaleable quantum computing is impossible.

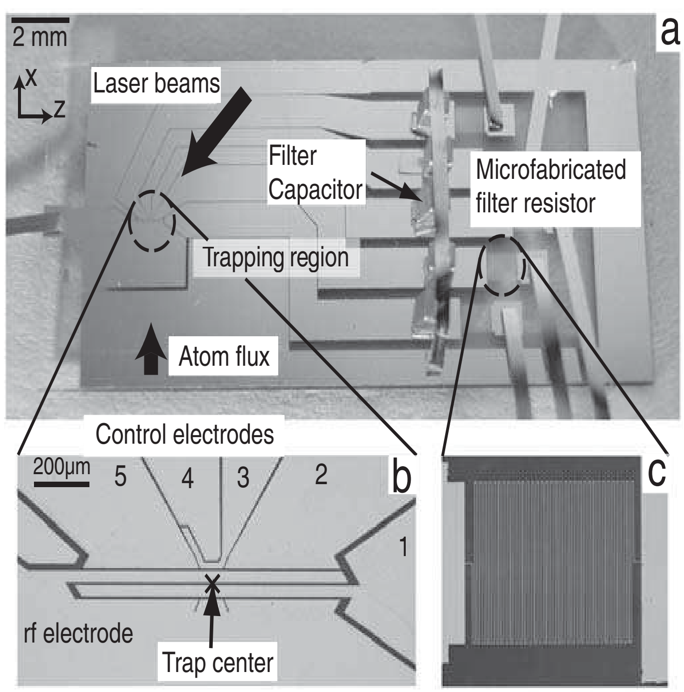




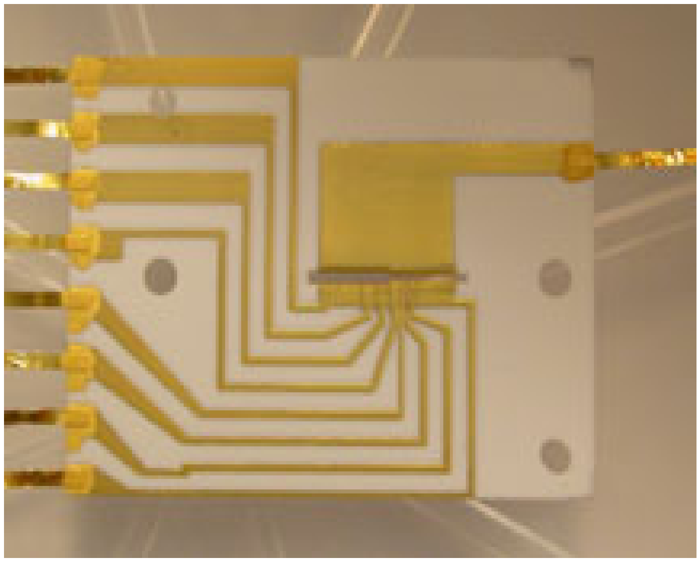
To get around this problem, recent experiments have concentrated on a new architecture: instead of a single trap, there will be many trapping regions, each holding a few ions. When a two-bit gate is performed, the ions holding the two q-bits are physically moved into the same trapping region; their normal mode is cooled into the ground state, and the gate is performed. They can then be moved back into a storage trap.

Trap-on-a-chip

These traps are also being dramatically shrunk down in size. Instead of the three-dimensional arrangement of electrodes used in earlier experiments, all of the electrodes in these new experiments are laid out on a flat surface, using photolithographic techniques. This allows many trapping regions to be established close to each other, so that ions can be transferred between them without their electronic states begin disturbed.



This layout was used for a recent experiment which demonstrated quantum teleportation between two ions in a trap.



Another important breakthrough was the use of “sympathetic cooling.” In addition to the ions used to store q-bits, additional ions of a different atomic species are included. The ions do not store q-bits, but because they have different resonance frequencies, they can be laser-cooled continuously without interfering with the other ions.

Performance

Ion trap experiments at present can handle three ions in a trap quite well—for certain special purposes, as many as twelve ions have been manipulated, though this is definitely not general-purpose. They can do hundreds of one- and two-bit gates before losing coherence. Progress is also being made in designs which permit parallel operations on many q-bits at once, which would make concatenated codes and fault-tolerant design possible. The gates at present are done with a fidelity of 95–99%. Ion traps are widely considered the mostly likely to succeed in building a medium-scale quantum computer in the not-too-distant future (if all goes well, within the next ten years). At present, ion traps are the leading contenders for scaleable quantum computing.

Next time: liquid-state NMR.