All About Interest

It might be said that business was invented when the concept of **interest** was invented. Interest is a percentage of a sum of money (the **principal**) that is borrowed or lent. The percentage is called the interest **rate**, and describes the proportion of the principal that accrues over a fixed time interval, called the **period**. Here are some typical interest bearing applications.

Home loan (now)	Principal of $450,000 with an annual interest rate of 5.75% per year
Home loan (1989)	Principal of $200,000 with an annual interest rate of 13.4% per year
Savings account (now)	Annual percentage rate of 1%
Savings account (1989)	4% per year in interest
Credit card account	Annual percentage rate of 18%
Certificate of deposit (now)	5.2% annual interest rate
Certificate of deposit (1989)	A rate of 9.8% per year

In cases of **simple** interest, computations are easy. The simple interest I on a principal of P dollars at an annual interest rate of r (expressed as a decimal) over t years is given by the formula $I = Prt$. For example, if you have a $1000 loan at 8% per annum of simple interest, then the interest for three years would be $1000 x 0.08 x 3 = $240.

Here are some exercises

1. Compute the annual interest that you would pay on a home loan of $375,000 at a typical rate for today, like 6.75%, with the amount that you would pay on the same loan if made in 1989 at a rate of 12.75%. Would it make good sense for you to refinance your home mortgage at this time if you were currently carrying a 12.75% loan?

2. You have $22,000 in your savings-plus account, and this account has an annual interest rate of 1.9%. You can get a one-year Certificate of Deposit (CD) that pays 4.8% through your credit union. How much more interest would you make in the first year if you moved your money from the savings account to the CD?

3. Suppose that your savings are **compounded** annually. This is different from simple interest and it means that each year the interest on your account is computed and is then added to the principal. So, if you had $1000 in the bank to begin with at 6% interest compounded annually and did not deposit any additional money, then after one year you would have the original $1000 plus 6% of $1000 in your account, for a total of $1060. The next year's interest would be computed on that amount rather than just on the original $1000. So after two years you would have $1060 + (0.06 x $1060), or $1123.60. Use this procedure of compounding annually to complete a table like the one below, but covering a period of 10 years starting with $5000 at an annual rate of 7.5%.

Year	Balance	Interest
0	$1000.00	$60.00
1	$1060.00	$63.60
2	$1123.60	•
•	•	•
•	•	•
•	•	•

It is actually kind of rare now to find interest compounded annually. Saving accounts will now typically compound monthly, or even daily. When you compound the interest over a period of less than one year, then the interest rate used in the compounding is the periodic interest rate, which is a fraction of the annual rate. So, if you have an annual rate of 4% compounded daily, then the periodic interest rate (here the daily interest rate) is $\frac{0.04}{365} \approx 0.000109589$. So if your original principal is $1000, then the first day you would earn $1000 x 0.000109589 = $0.109589, or almost eleven cents!

Another exercise

4. You have a credit card account that advertises an annual percentage rate (APR) of 16.5%. However, the interest on the account is compounded monthly. Compute the monthly percentage rate and use it to find out how much interest you would **ACTUALLY** pay on the original balance of $1000 over the course of one year. Assume that you made no new purchases and made no payments. What will be the **ACTUAL** interest rate on the account? (Hint: it will not be the same as the advertised rate.)

More information: If you start with a principal of P_0, an annual interest rate of r, and the interest is compounded n times per year, then the total principal after t years can be found using the formula $P(t) = P_0\left(1+\frac{r}{n}\right)^{nt}$.

An example:

You have a starting **principal** of $1000, and an annual interest **rate** of 3.6% compounded monthly. What is your balance after 2 years? The **annual rate** as a decimal is 0.036 and the **period** is one month, so the monthly interest rate is $\frac{0.036}{12} = 0.003$. The number of periods (months) in two years is 24, so the principal after two years is

$$P(24) = \$1000(1+0.003)^{24} \approx \$1074.54$$

Yet more exercises

5. Suppose that you start off with $20,000 invested in a 401K retirement plan, and that investment grows at a rate of 8%, compounded annually. How long will it be before you have $40,000 in the account? How long until you have $80,000? How long until $160,000? What do you notice about the time that it takes you money to double?

6. If you invested $40,000 in a retirement account at age 21, with an annual growth rate of 6.5%, compounded monthly, then **how much** will you have in your account when you retire at age 55?

7. Use graph paper to **make a graph** that has time (in years) on the horizontal axis and the amount of money that you have at each time on the vertical axis. Do this for n (years) ranging from 0 to 30, with a starting principal of $40,000 and a growth rate of 6.5%, compounded monthly. If you are doing this by hand, then you do not need to compute the amount for each year. Every five years would be fine. If you are doing this on the computer, then go ahead and find the value for each year. **Describe** as completely as possible how your money is growing. Is it a linear rate of growth? Quadratic? Something else …?

8. You have $40,000 to invest over a long period of time. Suppose that you can get an annual interest rate of 6.5% for this investment. **Compare** the amounts of money that you would make from your investment over a 30 year period if interest were compounded 1) annually, 2) monthly, 3) weekly, and 4) daily. **Be very careful about accuracy** here, especially in your computation and use of the periodic interest rate. Don't round numbers off or you lose too much accuracy. Make sure that your final answers are correct down to the nearest penny.

9. Is daily compounding the shortest time period that we can use? Discuss this question, and discuss the implications of shortening the compounding period. As usual, there is something very interesting to discover here, and if you want to get a good score on this assignment then you will need to give this question some serious thought and analysis.