MATH 125
Instructor: F. Bonahon
Computer Assignement 1
Due Thursday April 10 in the lab

The mathematical software Mathematica^{TM} is available on computers in KAP 262 (the computer lab of the Math Department), and in many other places on campus, such as King Hall, Waite Phillips (basement) or Leavey Library. Printing is free in the mathematics department, as long as the material is related to your math classes. There may be a printing charge at the other locations.

Most of what you will need to know is included below. However, if necessary, you can also find some on-line on Mathematica commands at the "Mathematica" page accessible from:
`http://college.usc.edu/mathematics/undergraduate/computinglabs.html`
The "Help" menu of the application is also a useful resource.

When asked to provide a print out, you should "sign" your print out by adding a comment line with your name and your student ID number. Mathematica's comments are to be inserted between brackets/stars `(*` and `*)`. For instance, you should type a line such as
`(* My name is Jane Doe and my Student ID number is 123-45-6789 *)`
next to what you intend to print (if your name is Jane Doe, of course). Mathematica will ignore this line. You can use cut and paste to put this line at all the locations where you will need to print, without having to retype everything. Make sure that each page of your print out is signed in this way.

To print, you can pull the command "Print" under the "File" menu. You can also select what you want to print and use the command "Print selection", still under the "File" menu, if there are portions of your file which you do not want to print.

Part 0.

Warm-up. We will use Mathematica as a calculator. No need to print anything. We are just playing with it.

Locate the application Mathematica on your computer, and open it. A window opens (after a pretty picture of a kind of spiny sphere, originally designed by a friend of mine!).

If, by any chance, no window opens, this means that Mathematica had already been opened by a previous user. Just pull "New" from the "File" menu, and then a window should open.

Type `2+3` , for instance. Locate the key "Enter" on your keyboard (not the key "Return"!) and press it.

In most cases, you will then have 20-60 seconds during which you can marvel at the power of modern technology, while waiting for an answer which even you can compute in your head without a calculator. Your computer is

actually not computing anything, but loading the program. After this first step, things will go much faster. Eventually, you get the answer `5`.

Now you can play with it. Compute 2×3 by typing `2*3` (and pressing "Enter"). Compute 2^{40} by typing `2^40`. Compute $\tan\frac{\pi}{3}$ by typing `Tan[Pi/3]` and pressing "Enter". Mathematica gives you the answer in the form $\sqrt{3}$. If you want a decimal approximation, use the function `N[ ]`. For instance, `N[ Tan[Pi/3] ]` will return the answer 1.73205.

In general, mathematical functions begin with a capital, such as `Sin`, `Cot`, `Log`, `Sqrt` (for square root) and use **square brackets** `[` and `]` (not parentheses `( )`) to enclose the quantities to which they apply. Multiplication is denoted by a star `*`, power by a hat `^` . The "Help" menu is a good place to find what you need, but guessing often works quite well. Round parentheses `( )` are used to group terms in the usual way, and are highly recommended to make sure that Mathematica really understands what you want. Spaces are irrelevant, and may help in making what you type more legible.

For instance, to compute $\sin\dfrac{3\pi}{\sqrt[3]{4\left(\sqrt{25}-3\right)}}$, you can type

```
Sin[(3*Pi) / ( ( 4*(Sqrt[25]-3) )^(1/3) ) ]
```

(Note that the answer is particularly simple).

No need to print anything yet. The real work begins in Part 1.

Part 1.

Now that we have suffered to learn how to compute derivatives, we will see how to instruct Mathematica to do the hard work for us. The Mathematica command to get the derivative of a function of the variable x is `D[ function , x]`, where the `x` indicates to Mathematica that x is the variable and not a constant. For instance, if you type

```
D[ x^3 + 5*x^2 , x ]
```

and press "Enter" (not "Return", remember), Mathematica returns

`10x + 3x^2`.

Use Mathematica to compute the derivative of the function $f(x) = x\cos^4\left(\sqrt[3]{2x+5}\right)$. As a hint, the Mathematica command for the function "cosine" is `Cos[ ]`, and you can use a fractional power for the root. Make sure that you use enough parentheses ( ). Print out your input line, and the Mathematica output; do not forget to include your signature line, as indicated at the beginning of this hand-out.

Part 2.

Mathematica is very good at graphing functions with a high precision. The command to graph the function `function` of the variable x, for x between a and b, is

```
Plot[ function, {x, a, b} ]
```

For instance,

```
Plot[ x^2, {x, -1, 5} ]
```

will produce a nice parabola. Try it.

Plot the function $f(x) = \sqrt{1-x^2}$ for x between 0 and 1 (you should get a circle arc). Print out your input and output, making sure that your signature line appears at least once on each page.

Part 3.

We saw in class that the tangent line of the curve of equation $y = f(x)$ at the point $(a, f(a))$ provides a very good approximation of the curve when x is near a. We will check this for the function $f(x) = \sqrt{1-x^2}$ at the point .55.

a) Use Mathematica to compute the derivative of $f(x) = \sqrt{1-x^2}$ (see Part 1), and to evaluate this derivative at $x = .55$. Mathematica uses a strange command to evaluate the function `function` of the variable x at the value `value`, namely

```
function /.  x-> value
```

Note that the funny arrow `->` is made up of a minus sign followed by the symbol "greater than". For instance,

```
x^2 /.  x-> 3
```

returns `9`.

Print out your input line, and the Mathematica output; do not forget to include your signature line.

b) Determine the equation for the line tangent to the curve of equation $y = \sqrt{1-x^2}$ at the point corresponding to $x = .55$. Just use pen and pencil, together with the answer from a). You do not need to print out or turn in anything here.

c) Plot the curve of equation $y = \sqrt{1-x^2}$ and its tangent line at $x = .55$ on the same graph, for values of x between 0 and 1. The command to plot on the same graph the two functions `function1` and `function2` for x between a and b is

```
Plot[ {function1, function2}, {x, a, b} ]
```

Print out your input line, and the Mathematica output; do not forget to include your signature line.

d) Zoom in by plotting the same two curves for x between .4 and .6. Print out your input line, and the Mathematica output; do not forget to include your signature line.

e) Same for x between .54 and .56. Print out your input line, and the Mathematica output; do not forget to include your signature line.

f) Same for x between .549 and .551. Print out your input line, and the Mathematica output; do not forget to include your signature line. What do you observe? (You can just write down your observation by hand on the print out.)