

**Math 125, Spring 2008**  
Practice exam for the second midterm

**Problem 1.** (10%) Find the derivative of the following functions.

- a)  $f(x) = \frac{1 + x^5}{1 - x^5}$   
 b)  $f(x) = \tan^3(2x + 5)$

**Problem 2.** (12%) If  $y$  is implicitly determined by the equation

$$x^4 + y^4 = 17$$

find the first derivative  $\frac{dy}{dx}$  and the second derivative  $\frac{d^2y}{dx^2}$  when  $x = 2$  and  $y = 1$ .

**Problem 3.** (12%) Find all local maxima and local minima of the function  $f(x) = x^4 - 4x^3 + 4x^2 + 1$ .

**Problem 4.** (20%) The first and second derivatives of the function

$$f(x) = \frac{2x^4 + 8x^3 + 8x^2 + x + 1}{x^2(x + 2)^2}$$

are  $f'(x) = \frac{-2(3x^2 + 6x + 4)}{x^3(x + 2)^3}$  and  $f''(x) = \frac{12(x + 1)(x^2 + 2x + 2)}{x^4(x + 2)^4}$ . (You do not need to check these computations.)

Sketch the graph of the function  $y = f(x)$ , and make sure that you indicate on the dotted lines below:

The equations of all horizontal asymptotes (if any): .....

The equations of all vertical asymptotes (if any): .....

The intervals where  $f$  is increasing (if any): .....

The  $x$ -coordinates of all inflection points (if any): .....

The intervals where  $f$  is concave upward (if any): .....

**Problem 5.** (12%) Find the point or points  $(x, y)$  of the parabola of equation  $y = x^2$  that are closest to the point  $(0, 1)$ .

**Problem 6.** (12%) A lighthouse is located  $\frac{1}{2}$  mile from the shore. Its light beam revolves at the rate of 1 revolution per minute. You are camping on the shore, 1 mile away from the point that is closest to the lighthouse. Assuming that the shore is completely straight, how fast will the illuminated part of the shore be moving as it passes over your tent?

**Problem 7.** (12%) a) Write the following limit as an integral.

$$\lim_{n \rightarrow \infty} \frac{2}{n} \sum_{i=1}^n \sqrt{4 - \left(\frac{2i}{n}\right)^2}$$

b) Compute the limit by interpreting the integral as an area.

**Problem 8.** (10%) Evaluate the following integrals. Make sure that you simplify your answer.

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a)  $\int \frac{x-1}{\sqrt{x}} dt$

b)  $\int_0^{\frac{\pi}{3}} \tan^2 x dx$