

Math 125, Fall 2006
Practice exam for the first midterm

Problem 1. Determine whether the following limits exist, finite or infinite. Explain your reasoning if the limit does not exist, or give the limit if it exists.

- (6%) $\lim_{x \rightarrow -\frac{1}{2}} \frac{x-1}{(2x+1)^2}$
- (6%) $\lim_{x \rightarrow 3} \frac{x}{\sqrt{x+1}-1}$
- (6%) $\lim_{x \rightarrow 0^+} \left(\frac{1}{x} - \frac{1}{|x|} \right)$
- (6%) $\lim_{x \rightarrow 0^-} \left(\frac{1}{x} - \frac{1}{|x|} \right)$
- (6%) $\lim_{x \rightarrow \frac{\pi}{4}} \tan 2x$
- (6%) $\lim_{x \rightarrow 1} \frac{1 - \sqrt{2x - x^2}}{(x-1)^2}$

Problem 2. Consider the function $f(x) = \frac{x^3}{x+1}$.

a) (6%) Can you use the Intermediate Value Theorem to conclude that the equation $f(x) = 1$ has a solution between 0 and +2? Explain.

b) (6%) Can you use the Intermediate Value Theorem to conclude that the equation $f(x) = 1$ has a solution between -2 and 0? Explain.

Problem 3.

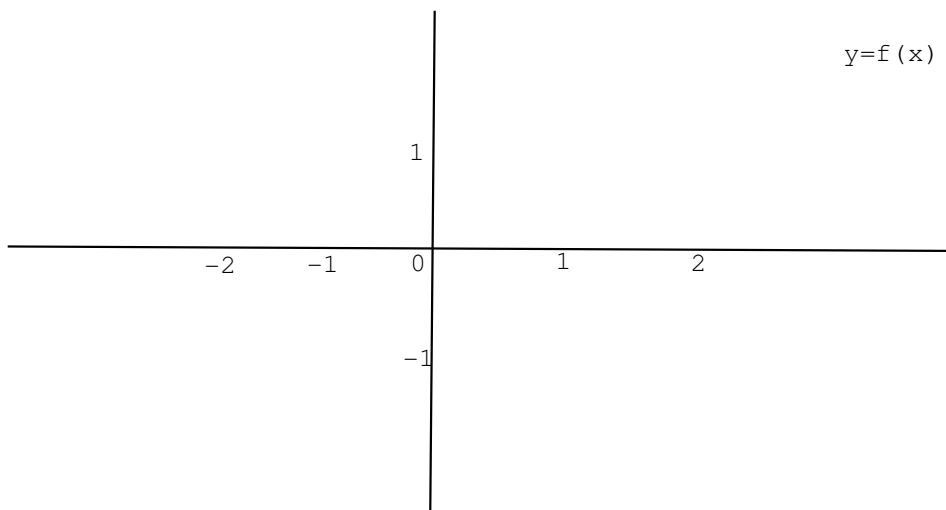
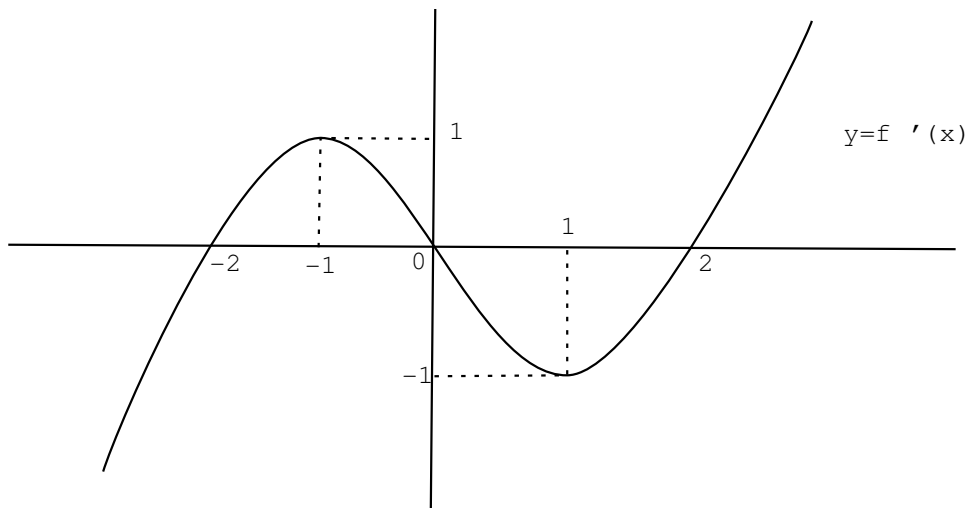
- (6%) Give a number δ such that

$$|(4x - 5) - 3| < .01 \text{ whenever } |x - 2| < \delta.$$

- (6%) Use the precise definition of limits to show that $\lim_{x \rightarrow 2} 4x - 5 = 3$.

Problem 4. (10%) Find the equation of the line tangent to the curve of equation $y = \frac{x^2 + 2}{\sqrt{x}}$ at the point $(1, 3)$.

Problem 5. (10%) The graph of the DERIVATIVE $f'(x)$ of a function $f(x)$ is indicated below. If, in addition, $f(0) = -1$, sketch a possible graph for the function $y = f(x)$. At which points is the line tangent to $y = f(x)$ horizontal? What is the point of steepest descent?



Problem 6. (10%) Express the following limit as the derivative of a certain function at a certain point, and use this property to compute the limit.

$$\lim_{x \rightarrow -\frac{2\pi}{3}} \frac{2 \cos x + 1}{x + \frac{2\pi}{3}}$$

Problem 7. (10%) Let $f(x) = \frac{1}{3 + \sqrt{x}}$. Use the original definition of the derivative as a limit (namely without using the product rule, power rule, etc. . .) to compute the derivative $f'(4)$.